# A New Criterion for Stability of Delayed Takagi-Sugeno Fuzzy Cohen-Grossberg Neural Networks

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## Abstract

In this paper, we derive a new criterion for the global asymptotic stability of Takagi-Sugeno(T-S) fuzzy Cohen-Grossberg neural networks with multiple time delays. By employing a more general type of fuzzy Lyapunov functional, we establish a easily verifiable delay independent stability condition that establishes a relationship between the system parameters of the delayed Takagi-Sugeno (T-S) fuzzy Cohen-Grossberg neural networks with respect to the nondecreasing and slope-bounded activation functions.

**Keywords**: Cohen-Grossberg Neural Networks, Time delays, Lyapunov Stability Theorems, T-S Fuzzy Systems.

## Introduction

In the recent years, many papers [1]-[6] have studied equilibrium and stability properties of Cohen-Grossberg neural networks (CGNNs) introduced in [7] as this class of neural networks has found important applications in the areas of pattern recognition, image and signal processing, parallel computation and control systems. In most of the applications of neural networks, the key point is to design a neural network possessing a globally asymptotically stable equilibrium point. Therefore, in the recent literature, stability of Cohen-Grossberg neural networks with or without delay parameters has received a great deal of attention and many different sufficient conditions for global asymptotic stability of the equilibrium point for delayed Cohen-Grossberg neural networks have been presented [1]-[7]. On the other hand, it has been shown in [8] that fuzzy logic theory may help to improve the desired behavior of complex dynamical systems and introduced the Takagi-Sugeno (T-S) fuzzy model. In [8], it has also been proved that T-S fuzzy systems can be used to transform a nonlinear system into a set of TS linear models. A T-S fuzzy system is essentially a nonlinear system described by a set of IF-THEN rules. Some certain nonlinear complex systems can be approximated by the overall fuzzy linear T-S models in order to conduct a investigation into the stability analysisof complex nonlinear systems. In a recent paper [9], by using Lyapunov stability theorems, somesufficient conditions for the stability the T-S fuzzy systems have been presented. The results obtained in [9] have led to many researchers studying the T-S fuzzy models to drive further stability results for various classes of fuzzy neural networks with time delays [10]-[19]. In this paper, wewilldealwiththestability of problem of Takagi-SugenofuzzyCohen-Grossbergneuralnetworkswithmultiple time delaysandobtain a newalternativecriterionforthe global asymptoticstability of theclass of delayedfuzzyCohen-Grossbergneuralnetworks.

**Stability Analysis of Takagi-Sugeno Fuzzy Cohen-Grossberg Neural Networks**

Consider the following Cohen-Grossberg neural networks with multiple time delay

$$\dot{x}_i(t) = d_i(x_i(t))\left[-c_i\big(x_i(t)\big) + \sum_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j\left(t - \tau_{ij}\right)\right) + u_i\right] \tag{1}$$

where $n$ is the number of the neurons in the neural system, $x_i$ denotes the state of the $i$th neuron, $d_i(x_i)$ represent the amplification functions, and $c_i(x_i)$ represent the behaved functions. The constants $a_{ij}$ are the interconnection parameters of the neurons within the neural system, the constants $b_{ij}$ are interconnection parameters of the neurons with time delay parameters $\tau_{ij}$. The $f_i(.)$ denote the activation functions of neurons. The constants $u_i$ are some external inputs. In system (1), $\tau_{ij} \geq 0$ are constant time delays with $\tau = \max(\tau_{ij}), 1 \leq i, j \leq n$. Accompanying the neural system (1) is an initial condition of the form: $x_i(t) = \phi_i(t) \in C([-\tau, 0]), R)$, where $C([-\tau, 0]), R)$ denotes the set of all continuous functions from $[-\tau, 0]$ to $R$.

The assumptions on the functions $d_i(x)$, $c_i(x)$ and $f_i(x)$ in (1) are defined to be as follows:
$H_1$: The functions $d_i(x), (i = 1, 2, \dots, n)$ satisfy the conditions

$$0 < \mu_i \leq d_i(x) \leq \rho_i, \forall x \in R$$

where $\mu_i$ and $\rho_i$ are some positive constants.
$H_2$: The functions $c_i(x), (i = 1, 2, \dots, n)$ satisfy the conditions

$$\frac{c_i(x) - c_i(y)}{x - y} = \frac{|c_i(x) - c_i(y)|}{|x - y|} \geq \gamma_i > 0, i = 1, 2, \dots, n, \forall x, y \in R, x \neq y$$

where $\gamma_i$ are some positive constants.
$H_3$: The functions $f_i(x), (i = 1, 2, \dots, n)$ satisfy the conditions

$$|f_i(x) - f_i(y)| \leq \ell_i |x - y|, i = 1, 2, \dots, n, \forall x, y \in R, x \neq y$$

where $\ell_i$ are some positive constants.

In order to simplify the proofs, the equilibrium point $x^*$ of Cohen-Grossberg neural network model (1) can be transformed to the origin. Using the transformation $z(t) = x(t) - x^*$, we can transform system (1) into a new system of the form :

$$\dot{z}_i(t) = \alpha_i(z_i(t))\left[-\beta_i\big(z_i(t)\big) + \sum_{j=1}^{n} a_{ij} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij} g_j\left(z_j\left(t - \tau_{ij}\right)\right)\right] \tag{2}$$

where the following properties hold:

$$\alpha_i\big(z_i(t)\big) = d_i(z_i(t) + x_i^*), i = 1, 2, \dots, n$$
$$\beta_i\big(z_i(t)\big) = c_i(z_i(t) + x_i^*) - c_i(x_i^*), i = 1, 2, \dots, n$$
$$g_i\big(z_i(t)\big) = f_i(z_i(t) + x_i^*) - f_i(x_i^*), i = 1, 2, \dots, n$$

Since $x(t) \to x^*$ as $z(t) \to 0$, establishing stability of the origin of system (2) will be the main objective.

In [29], the T-S fuzzy Cohen-Grossberg neural network with multiple time delays is defined by the following mathematical model :
**Plant Rule** r:
**IF**$\{\theta_1(t) \ is \ M_{r1}\}$ and … and $\{\theta_p(t) \ is \ M_{rp}\}$
**THEN**

$$\dot{z}_i(t) = \alpha_i^{(r)}(z_i(t))\left[-\beta_i^{(r)}(z_i(t)) + \sum_{j=1}^{n} a_{ij}^{(r)} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij}^{(r)} g_j\left(z_j(t-\tau_{ij})\right)\right] \qquad (3)$$

where $\theta_l(t)$ $(l = 1, 2, \ldots, p)$ are the premise variables. $M_{rl}$ $(r \in \{1, 2, \ldots, m\}, l \in \{1, 2, \ldots, p\}$ are the fuzzy sets and $m$ is the number of **IF-THEN** rules.

By inferring from the fuzzy models, (3) are stated as follows [19] :

$$\dot{z}_i(t) = \sum_{r=1}^{m} h_r(\theta(t))\left\{\alpha_i^{(r)}(z_i(t))\left[-\beta_i^{(r)}(z_i(t)) + \sum_{j=1}^{n} a_{ij}^{(r)} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij}^{(r)} g_j\left(z_j(t-\tau_{ij})\right)\right]\right\} \quad (4)$$

where $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_p(t)]^T$, $\omega_r(\theta(t)) = \prod_{l=1}^{p} M_{rl}(\theta_\ell(t))$ and $h_r(\theta(t)) = \frac{\omega_r(\theta(t))}{\sum_{r=1}^{m} \omega_r(\theta(t))}$ denote the weight and averaged weight of each fuzzy rule, respectively. The term $\omega_{rl}(\theta_l(t))$ is the grade membership of $\theta_l(t)$ in $\omega_{rl}$. It is assumed that $\omega_r(\theta(t)) \geq 0, r \in \{1, 2, \ldots, m\}$. Therefore, it follows that $\sum_{r=1}^{m} h_r(\theta(t)) = 1$ for all $t \geq 0$.

For the model of T-S fuzzy neural system (4), the assumptions $H_1$, $H_2$ and $H_3$ are now respectively formulated as follows:

$$0 < \alpha_i^{(r)}(z_i(t)) \leq \rho_i^{(r)}, i = 1, 2, \ldots, n$$
$$z_i(t)\beta_i^{(r)}(z_i(t)) \geq \gamma_i^{(r)} z_i^2(t) \geq 0, i = 1, 2, \ldots, n$$
$$|g_i(z_i(t))| \leq k_i|z_i(t)|, z_i(t)g_i(z_i(t)) \geq 0, i = 1, 2, \ldots, n$$

**Stability of Delayed Takagi-Sugeno Fuzzy Cohen-Grossberg Neural Networks**

In this section, we obtain the following stability result :

**Theorem 1:** Under Assumptions $H_1$, $H_2$ and $H_3$, the origin of the delayed T-S fuzzy Cohen-Grossberg neural networks defined by (4) is globally asymptotically stable if the following condition holds:

$$\delta = \frac{2\mu\gamma}{\rho} K^{-1} - \sum_{r=1}^{m}(|A_r| + |A_r^T|) - 2\sum_{r=1}^{m}\sqrt{\|B_r\|_1\|B_r\|_\infty} I > 0$$

where $K = diag(k_1, k_2, \ldots, k_n) \mu = min\{\mu_i^{(r)}\}, \rho = max\{\rho_i^{(r)}\}$ and $\gamma = max\{\gamma_i^{(r)}\}, \ell = max\{\ell_i\}$, $i = 1, 2, \ldots, n, r = 1, 2, \ldots, m$, $A_r = (a_{ij}^{(r)})_{nxn}, |A_r| = (|a_{ij}^{(r)}|)_{nxn}$ and $B_r = (b_{ij}^{(r)})_{nxn}$.

**Proof:** Consider the Lyapunov functional:

$$V(z(t)) = 2\sum_{i=1}^{n}\int_0^{z(t)} s\,ds + 2\sigma\sum_{i=1}^{n}\int_0^{z(t)} g_i(s)\,ds + \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t} g_j^2(z_j(\zeta))\,d\zeta$$
$$+ \sigma\rho\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t}\left|b_{ij}^{(r)}\right| g_j^2(z_j(\zeta))\,d\zeta$$

where the $\xi_r, \varepsilon$ and $\sigma$ are some positive constants to be determinated later. Calculating the time derivative of $V(x(t))$ along the trajectories of system (4) yields

$$\dot{V}(z(t)) = 2\sum_{i=1}^{n} z_i(t)\dot{z}_i(t) + 2\sigma\sum_{i=1}^{n} g_i(z_i(t))\dot{z}_i(t) + \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} g_j^2(z_j(t-\tau_{ij})) + \sigma\rho\sum_{r=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n}\xi_r g_j^2(z_j(t))$$
$$- \sigma\rho\sum_{r=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n}\xi_r g_j^2(z_j(t-\tau_{ij}))$$

$$= -2\sum_{r=1}^{m} h_r\big(\theta(t)\big) \sum_{i=1}^{n} z_i(t)\alpha_i^{(r)}\big(z_i(t)\big)\beta_i^{(r)}\big(z_i(t)\big)$$

$$+2\sum_{r=1}^{m} h_r\big(\theta(t)\big) \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^{(r)}(z_i(t))a_{ij}^{(r)} z_i(t)g_j(z_j(t))$$

$$+2\sum_{r=1}^{m} h_r\big(\theta(t)\big) \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^{(r)}(z_i(t))b_{ij}^{(r)} z_i(t)g_j\big(z_j\big(t-\tau_{ij}\big)\big)$$

$$-2\sigma\sum_{r=1}^{m} h_r\big(\theta(t)\big) \sum_{i=1}^{n} \alpha_i^{(r)}(z_i(t))\beta_i^{(r)}(z_i(t))g_i(z_i(t))$$

$$+2\sigma\sum_{r=1}^{m} h_r\big(\theta(t)\big) \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^{(r)}(z_i(t))a_{ij}^{(r)} g_i(z_i(t))g_j(z_j(t))$$

$$+2\sigma\sum_{r=1}^{m} h_r\big(\theta(t)\big) \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^{(r)}(z_i(t))b_{ij}^{(r)} g_i(z_i(t))g_j\big(z_j\big(t-\tau_{ij}\big)\big)$$

$$+\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} g_j^2(z_j(t)) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} g_j^2\big(z_j\big(t-\tau_{ij}\big)\big)$$

$$+\sigma\rho\sum_{r=1}^{m}\xi_r \sum_{i=1}^{n}\sum_{j=1}^{n} \big|b_{ij}^{(r)}\big| g_j^2(z_j(t)) - \sigma\rho\sum_{r=1}^{m}\xi_r \sum_{i=1}^{n}\sum_{j=1}^{n} \big|b_{ij}^{(r)}\big| g_j^2\big(z_j\big(t-\tau_{ij}\big)\big) \qquad (5)$$

Under the Assumptions $H_1$, $H_2$ and $H_3$, in [29], the following inequalities have been shown to be held:

$$-2\sum_{r=1}^{m} h_r\big(\theta(t)\big)\sum_{i=1}^{n} z_i(t)\alpha_i^{(r)}\big(z_i(t)\big)\beta_i^{(r)}\big(z_i(t)\big) \le -2\mu\gamma\sum_{i=1}^{n} z_i^2(t) \qquad (6)$$

$$2\sum_{r=1}^{m} h_r\big(\theta(t)\big)\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^{(r)}(z_i(t))a_{ij}^{(r)} z_i(t)g_j(z_j(t)) \le \mu\gamma\sum_{i=1}^{n} z_i^2(t) + n\upsilon\sum_{i=1}^{n} g_i^2(z_i(t)) \qquad (7)$$

$$2\sum_{r=1}^{m} h_r\big(\theta(t)\big)\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^{(r)}(z_i(t))b_{ij}^{(r)} z_i(t)g_j\big(z_j\big(t-\tau_{ij}\big)\big) \le 2\mu\gamma\sum_{i=1}^{n} z_i^2(t) + \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} g_j^2\big(z_j\big(t-\tau_{ij}\big)\big) \qquad (8)$$

$$-2\sigma\sum_{r=1}^{m} h_r\big(\theta(t)\big)\sum_{i=1}^{n}\alpha_i^{(r)}\big(z_i(t)\big)\beta_i^{(r)}\big(z_i(t)\big)g_i\big(z_i(t)\big) \le -2\sigma\mu\gamma\big|g^T\big(z(t)\big)\big| + K^{-1}\big|g\big(z(t)\big)\big| \qquad (9)$$

$$2\sigma\sum_{r=1}^{m} h_r\big(\theta(t)\big)\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^{(r)}(z_i(t))a_{ij}^{(r)} g_i(z_i(t))g_j(z_j(t)) \le \sigma\rho\sum_{r=1}^{m}\big(|A_r| + |A_r^T|\big)\big|g\big(z(t)\big)\big| \qquad (10)$$

where

$$\upsilon = \frac{mna^2\rho^2}{\mu\gamma},\, \varepsilon = \frac{mnb^2\rho^2}{\mu\gamma},$$

and $a$ and $b$ are some positive constant such that $\left|a_{ij}^{(r)}\right| \leq a$ and $\left|b_{ij}^{(r)}\right| \leq b$ for all, $i, j = 1, 2, \ldots, n$ and $r = 1, 2, \ldots, m$

Wenotethefollowinginequality

$$2\sigma \sum_{r=1}^{m} h_r(\theta(t)) \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i^{(r)}(z_i(t)) b_{ij}^{(r)} g_i(z_i(t)) g_j\left(z_j\left(t - \tau_{ij}\right)\right)$$

$$\leq 2\sigma \sum_{r=1}^{m} h_r(\theta(t)) \sum_{i=1}^{n} \sum_{j=1}^{n} \left|\alpha_i^{(r)}(z_i(t))\right| \left|b_{ij}^{(r)}\right| |g_i(z_i(t))| \left|g_j\left(z_j\left(t - \tau_{ij}\right)\right)\right|$$

$$\leq \sigma \sum_{r=1}^{m} h_r(\theta(t)) \rho \sum_{i=1}^{n} \sum_{j=1}^{n} 2\left|b_{ij}^{(r)}\right| |g_i(z_i(t))| \left|g_j\left(z_j\left(t - \tau_{ij}\right)\right)\right|$$

$$\leq \sigma \sum_{r=1}^{m} h_r(\theta(t)) \rho \left\{ \frac{1}{\xi_r} \sum_{i=1}^{n} \sum_{j=1}^{n} \left|b_{ij}^{(r)}\right| |g_i^2(z_i(t))| + \xi_r \sum_{i=1}^{n} \sum_{j=1}^{n} \left|b_{ij}^{(r)}\right| |g_j^2\left(z_j\left(t - \tau_{ij}\right)\right)| \right\}$$

$$\leq \sigma \rho \left\{ \sum_{r=1}^{m} \frac{1}{\xi_r} \sum_{i=1}^{n} \sum_{j=1}^{n} \left|b_{ij}^{(r)}\right| |g_i^2(z_i(t))| + \sum_{r=1}^{m} \xi_r \sum_{i=1}^{n} \sum_{j=1}^{n} \left|b_{ij}^{(r)}\right| |g_j^2\left(z_j\left(t - \tau_{ij}\right)\right)| \right\} \tag{11}$$

Using (6)-(11)$\dot{V}(z(t))$ yields

$$\dot{V}(z(t)) \leq nv \sum_{j=1}^{n} g_i^2(z_i(t)) + n\varepsilon \sum_{j=1}^{n} g_i^2(z_i(t)) - 2\sigma\mu\gamma \left|g^T(z(t))\right| K^{-1} |g(z(t))|$$

$$+ \sigma\rho \sum_{r=1}^{m} \left|g^T(z(t))\right|(|A_r| + |A_r^T|) |g(z(t))|$$

$$+ \sigma\rho \sum_{r=1}^{m} \frac{1}{\xi_r} \sum_{i=1}^{n} \sum_{j=1}^{n} \left|b_{ij}^{(r)}\right| |g_i^2(z_i(t))| + \sigma\rho \sum_{r=1}^{m} \xi_r \sum_{i=1}^{n} \sum_{j=1}^{n} \left|b_{ij}^{(r)}\right| |g_i^2(z_i(t))|$$

$$\leq n(v + \varepsilon)\left|g^T(z(t))\right| |g(z(t))| - 2\sigma\mu\gamma \left|g^T(z(t))\right| K^{-1} |g(z(t))|$$

$$+ \sigma\rho \sum_{r=1}^{m} \left|g^T(z(t))\right|(|A_r| + |A_r^T|) |g(z(t))|$$

$$+ \sigma\rho \sum_{r=1}^{m} \frac{1}{\xi_r} \|B_r\|_\infty \left|g^T(z(t))\right| |g(z(t))| + \sigma\rho \sum_{r=1}^{m} \xi_r \|B_r\|_1 \left|g^T(z(t))\right| |g(z(t))|$$

Let

$$\xi_r = \frac{\sqrt{\|B_r\|_\infty}}{\sqrt{\|B_r\|_1}}, r = 1, 2, \ldots, m$$

Then, we have

$$\dot{V}(z(t)) \leq n(v + \varepsilon)\left|g^T(z(t))\right| |g(z(t))| - 2\sigma\mu\gamma \left|g^T(z(t))\right| K^{-1} |g(z(t))|$$

$$+ \sigma\rho \sum_{r=1}^{m} \left|g^T(z(t))\right|(|A_r| + |A_r^T|) |g(z(t))|$$

$$+2\sigma\rho\sum_{r=1}^{m}\sqrt{\|B_r\|_1\|B_r\|_\infty}|g^T(z(t))||g(z(t))|$$

$$= n(v+\varepsilon)|g^T(z(t))||g(z(t))|$$

$$-\sigma\rho|g^T(z(t))|(\frac{2\mu\gamma}{\rho}K^{-1}-\sum_{r=1}^{m}(|A_r|+|A_r^T|)-2\sqrt{\|B_r\|_1\|B_r\|_\infty})|g(z(t))|$$

$$= n(v+\varepsilon)|g^T(z(t))||g(z(t))|-\sigma\rho|g^T(z(t))|\Omega|g(z(t))|$$

The fact that $\Omega$ is a positive definite matrix implies that

$$\dot{V}(z(t)) \leq n(v+\varepsilon)\|g(z(t))\|_2^2 - \sigma\rho\lambda_{min}(\Omega)\|g(z(t))\|_2^2$$

$$= (n(v+\varepsilon) - \sigma\rho\lambda_{min}(\Omega))\|g(z(t))\|_2^2$$

For the choice $\sigma > \frac{n(v+\varepsilon)}{\rho\lambda_{min}(\Omega)}$, $\dot{V}(z(t)) < 0$ for all $g(z(t)) \neq 0$. Let $g(z(t)) = 0$. Then, $\dot{V}(z(t))$ is of the form:

$$\dot{V}(z(t)) = -2\sum_{r=1}^{m}h_r(\theta(t))\sum_{i=1}^{n}z_i(t)\alpha_i^{(r)}(z_i(t))\beta_i^{(r)}(z_i(t))$$

$$+2\sum_{r=1}^{m}h_r(\theta(t))\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^{(r)}(z_i(t))b_{ij}^{(r)}z_i(t)g_j(z_j(t-\tau_{ij}))$$

$$-\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}g_j^2(z_j(t-\tau_{ij})) + \sigma\rho\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right|g_j^2\left(z_j(t-\tau_{ij})\right) \quad (12)$$

which, when combined with (6) and (8) in (12), results in

$$\dot{V}(z(t)) = -2\mu\gamma\sum_{i=1}^{n}z_i^2(t) + \mu\gamma\sum_{i=1}^{n}z_i^2(t) + \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}g_j^2(z_j(t-\tau_{ij}))$$

$$-\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}g_j^2(z_j(t-\tau_{ij})) - \sigma\rho\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right|g_j^2\left(z_j(t-\tau_{ij})\right)$$

$$= -\mu\gamma\sum_{i=1}^{n}z_i^2(t) - \sigma\rho\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right|g_j^2\left(z_j(t-\tau_{ij})\right) \leq -\mu\gamma\sum_{i=1}^{n}z_i^2(t)$$

It is easy to see that $\dot{V}(z(t)) < 0$ for all $z(t) \neq 0$. Now let $g(z(t)) = 0$ and $z(t) = 0$. Then

$$\dot{V}(z(t)) = -\sigma\rho\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right|g_j^2\left(z_j(t-\tau_{ij})\right)$$

Note that if $g_j\left(z_j(t-\tau_{ij})\right) \neq 0$ for any pairs of $i$ and $j$, then $\dot{V}(z(t)) < 0$. Hence, it follows that $\dot{V}(z(t)) = 0$ if and only if $g(z(t)) = 0$, $z(t) = 0$ and $g_j\left(z_j(t-\tau_{ij})\right) = 0$ for all $i$ and $j$ and $\dot{V}(z(t)) < 0$ in all other cases. It is easy to verify that $V(z(t))$ is radially unbounded since $V(z(t)) \to \infty$ as $\|z(t)\| \to \infty$. Therefore, we can directly conclude that the origin of the T-S fuzzy Cohen-Grossberg neural network model (4) is globally asymptotically stable.

## Conclusions

This paper has presented a new delay independent sufficient condition for the global asymptotic stability of delayed Takagi-Sugeno (T-S) fuzzy Cohen-Grossberg neural networks by using a class of fuzzy Lyapunov functional with respect to the nondecreasing and slope-bounded activation functions. This stability condition can be easily checked as it is completely expressed in terms of the network parameters of the neural system. It is also shown that the stability criterion obtained in this paper for delayed Takagi Sugeno (T-S) fuzzy Cohen-Grossberg neural networks improves and generalizes a recently published corresponding stability result.

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